Thermal Stratification Modelling of Small Shallow Aquaculture Ponds

Noel P Merrick
National Centre for Groundwater Management, University of Technology, Sydney, PO Box 123, Broadway, NSW 2007

Jiacai Gao
Manly Hydraulics Laboratory, Department of Public Works and Services, 110B King Street, Manly Vale, NSW 2093

Abstract A mathematical model has been developed to simulate temporal variations and vertical stratification of water temperature in small and shallow aquaculture ponds. The dynamic, mechanistic model calculates the water temperature of ponds in discrete, homogeneous horizontal layers. Energy exchanges between the pond and atmosphere can be estimated accurately with theoretical and empirical relationships commonly applied to heat balance calculations in other larger and deeper bodies of water. Energy transfer between the layers caused by turbulent mixing is simulated by using an integral energy model. Water-sediment energy flux is included in the model. The diffusion coefficient in the epilimnion is computed as a function of wind speed only. The thermocline and hypolimnion diffusion coefficients are related to the Brunt-Vaisala frequency. The model uses a fully implicit central difference scheme to discretise the partial differential equation for a control volume. The time step of one hour and distance step of 2 cm are sufficient to obtain accurate results. The model runs for 24 hours. This approach is an original development and has not been applied previously to aquaculture ponds. Measured temperature profiles from both freshwater and salt-water ponds were used to calibrate and validate the developed model. The average regression coefficients for model calibration and validation are 0.93 and 0.84, respectively. The maximum simulation errors are less than 1.7 °C for both calibration and validation runs. The model can predict thermal stratification events within 1-3 hours, and the maximum daily stratification magnitude with errors less than 1.0 °C. The model is quite general in application, as evidenced by successful application to a number of ponds after calibration at different ponds in different locations. Sensitivity analyses show that the model is most sensitive to solar radiation and wind speed, and is sensitive to extinction coefficient, the maximum hypolimnion diffusion coefficient, air temperature and relative humidity. The model is insensitive to wind sheltering coefficient, radiation surface absorption factor, epilimnion diffusion coefficient and water-sediment energy flux.

1. INTRODUCTION

The mathematical modelling of aquaculture ponds has been slow to develop due to a lack of data and understanding of the basic principles governing the complex physical and biological interactions taking place within the system [Bolte et al., 1986]. As a result, many early efforts in pond water quality modelling fall into the category of empirical or "black box" models. Only recently have "mechanistic" or "internally descriptive" models been developed for use in aquaculture. Mechanistic models are based on sets of mathematical descriptions of the processes that occur within the modelled system [Piedrahita, 1988]. General principles of computer modelling as they apply to the modelling and simulation of aquaculture systems have been reviewed by Bolte et al. [1986], Marjanovic and Orlob [1986], Losordo [1988], Cuenco [1989] and Piedrahita [1991].

Almost all aquaculture pond models assume the conditions in the ponds to be uniform for the modelling purpose [Krant et al. ,1982; de Jager and Walmsley, 1984; Klemetson and Rogers,1985; Gao and Merrick, 1996]. Most authors, however, acknowledge the fact that the ponds are in fact not homogeneous, and that stratification might play a significant role in determining oxygen concentration. The magnitude of temperature and dissolved oxygen (DO) stratification noted by Losordo [1988], Chang and Ouyang [1988] and Gao [1997], suggests that this effect must be explicitly accounted for to obtain accurate prediction of temperature and DO concentration. The only documented aquaculture pond stratification models are those of Cathcart and Wheaton [1987], Losordo and Piedrahita [1991], Chang and Ouyang [1988] and Gao [1997].

This paper reports the development and application of an integral energy model of temperature stratification for shallow and small aquaculture ponds.

2. MODEL DEVELOPMENT

A general form of the heat equation for a water body with a horizontal area dependent on depth is given by [Henderson-Sellers, 1984; Hondzo and Stefan, 1991]:

$$\frac{\partial T}{\partial t}=\frac{1}{A}\frac{\partial}{\partial z}\left[AK\frac{\partial T}{\partial z}\right]-\frac{1}{\rho cA}\frac{\partial(\phi A)}{\partial z}$$

where T is water temperature (°C), t is time (h), A is horizontal area (m^2), K is turbulent diffusion coefficient (m^2h^{-1}), ϕ is direct radiative heat flux (kJ m^{-2} h^{-1}), ρ is density of water (kgm^{-3}), c is heat capacity of water (kJ kg^{-1} $°C^{-1}$), assumed to be constant in the model.

The vertical temperature profile in a pond is computed from a balance between incoming heat from solar and long-wave radiation and the outflow of heat through convection, evaporation, and back radiation. The net income in heat results in an increase in water temperature. The heat balance equation is given by [WRE, 1968; Riley, 1988; Losordo and Piedrahita, 1991; Gao and Merrick, 1996]:

$$\phi_{net} = \phi_{sn} + \phi_{at} - \phi_{ws} - \phi_e - \phi_c$$

where ϕ_{net} is net heat flux passing the air-water interface ($kJm^{-2}h^{-1}$), ϕ_{sn} is net solar radiation below water surface ($kJm^{-2}h^{-1}$), ϕ_{at} is atmospheric radiation ($kJm^{-2}h^{-1}$), ϕ_{ws} is water surface radiation ($kJm^{-2}h^{-1}$), ϕ_e is evaporative heat loss ($kJm^{-2}h^{-1}$), ϕ_c is sensible heat transfer ($kJm^{-2}h^{-1}$).

2.1 Energy Balance Consideration

Solar radiation below water surface The net solar irradiance below the water surface is calculated as [Riley, 1988; Hondzo and Stefan, 1991]:

$$\phi_{sn} = (1-R)\,(1-\beta)\phi_s$$

where R is reflectivity of a smooth water surface, β is surface absorption fraction, and ϕ_s is incoming solar radiation ($kJm^{-2}h^{-1}$).

Solar radiation attenuation The attenuation of solar radiation with depth follows Beers law [Laska, 1981; Stefan et al., 1983; Lund and Keinonen, 1984]:

$$\phi_{sn,\,i+1} = \phi_{sn,\,i}\exp(-\eta\Delta z_i)$$

where $\phi_{sn,\,i}$ is solar radiation at the top of the layer i ($kJm^{-2}h^{-1}$), $\phi_{sn,\,i+1}$ is solar radiation at the bottom of the layer i ($kJm^{-2}h^{-1}$), η is extinction coefficient (m^{-1}), and Δz_i is thickness of the layer (m).

Atmospheric radiation The amount of long wave radiation penetrating the water surface is called the net atmospheric radiation and can be calculated as [WRE, 1968; Octavio et al., 1977]:

$$\phi_{at} = (1-r)e\sigma(T_{ak})^4$$

where r is reflectance of water surface to long-wave radiation,$\approx$0.03, σ is Stefan-Boltzman constant, 2.04×10^{-7} ($kJm^{-2}h^{-1}\,{}^{\circ}K^{-4}$), T_{ak} is absolute air temperature 2 m above the water surface (K), e is average emittance of the atmosphere, $0.398\times10^{-5}\,(T_{ak})^{2.148}$

Water surface radiation The loss of energy from a water body by long-wave radiation can be estimated as [Octavio et al., 1977; Henderson-Sellers, 1984]:

$$\phi_{ws} = 0.97\sigma(T_{wk})^4$$

where T_{wk} is absolute water temperature (K).

Evaporative heat loss The evaporative heat flux for small ponds can be estimated as [Fritz et al., 1980; Orlob, 1981]:

$$\phi_e = NW_2(e_s-e_a)$$

where N is an empirical coefficient derived from the Lake Hefner formulation, 5.0593 ($kJm^{-2}km^{-1}mmHg^{-1}$), W_2 is wind speed at a reference height 2 m above the water surface (kmh^{-1}), e_s is saturated vapour pressure at T_{wk} (mmHg), e_a is vapour pressure above the pond surface (mm Hg).

Sensible heat transfer The sensible heat transfer to or from a small water body can be estimated as [Fritz et al., 1980; Orlob, 1983]:

$$\phi_c = 1.5701W_2(T_{wc}-T_{ac})$$

where T_{wc} and T_{ac} are water and air temperatures ($^{\circ}C$).

Sediment energy transfer The heat exchange between water and sediment is simulated with the assumption that the sediment temperature at infinite depth below the pond bottom is unchanged, i.e. the bottom boundary condition is taken as [Henderson-Sellers, 1984]:

$$T(z,t)\,(z\rightarrow\infty) = T_S$$

2.2 Temperature Calculation

The turbulent kinetic energy (TKE) available for possible entrainment at the interface is estimated by [Ford and Stefan, 1980; Harleman, 1982; Riley, 1988]:

$$TKE = W_{STR}\tau_o u_* A_s\Delta t = W_{STR}\rho_w u_*^3 A_s\Delta t$$

where TKE is turbulent kinetic energy (J), u_* is wind induced water shear velocity (ms^{-1}), A_s is surface area (m^2), τ_o is wind shear stress at the water surface (Nm^{-2}), Δt is time step (s), u_w is wind velocity above water surface (ms^{-1}), ρ_w is water density (kgm^{-3}), ρ_a is air density (kgm^{-3}), W_{STR} is wind sheltering coefficient, representing the fraction of a pond subject to wind energy.

The buoyant potential energy (BPE) of the mixed layer can be thought of as the potential work required to produce a change in the depth of the mixed layer by entraining denser water from below. The BPE is given as [Ford and Stefan, 1980; Harleman, 1982; Riley, 1988]:

$$BPE = V_m\Delta\rho g(z_m-z_g)$$

where BPE is potential energy (J), V_m is volume of the mixed layer (m^3), $\Delta\rho$ is density difference between the mixed layer and the layer immediately below the mixed layer (kgm^{-3}), z_m is depth of the mixed layer (m), and z_g is depth of the centre of gravity of the mixed layer (m).

The depth of wind mixing is determined from a stability consideration [Ford and Stefan, 1980; Harleman, 1982; Riley, 1988] given by:

$$\sigma = \frac{TKE}{BPE} = \frac{W_{STR}\rho_w u_*^3 A_S \Delta t}{V_m \Delta\rho g(z_m - z_g)}$$

The mixing computation proceeds from layer to layer entraining one layer at a time and computing the corresponding V_m, $\Delta\rho$, z_m, and z_g until the stability criterion (σ) is less than unity.

The calculation of temperature profiles in the integral energy model is accomplished in two steps: (1) During each time interval (Δt) the temperature profile is calculated from estimating and assuming heat energy inputs and losses; (2) In the second step, σ is evaluated for that time step. If σ is less than one, the mixed layer depth (D_m) is left unchanged and the temperature profile calculated in the first step is utilised for initialization of the next time interval temperature calculations. However, if $\sigma \geq 1$, the thickness of the mixed layer is increased by Δz ($D_m + \Delta z$), and the stability criterion is evaluated again. The surface heat balance, temperature and density of the mixed layer and of the other horizontal layers must be recalculated as the mixed layer depth is changed. This process is continued in the time interval until the ratio σ is less than 1. At that point, the mixed layer depth is set, and the temperature profile is recalculated.

During periods of heating of the water body, turbulent diffusion is the dominant source of vertical mixing (Octavio et al., 1977). Most authors using one-dimensional diffusion models have assumed that vertical advective and convective transport are minimal under stratified conditions. However, during periods of net cooling of water bodies, convective transport due to buoyancy instabilities in the water column becomes a dominant source of vertical mixing. Penetrative convection due to density instability is simulated with a "numerical mixing" process in the model. When the heat flow computation results in the "unnatural" situation of a warmer layer of water being below a cooler layer, then the temperature of the two layers is averaged. The process of numerical mixing will result in the averaging of mass concentrations in the water column.

2.3 Numerical Scheme

A fully implicit central difference scheme is used to discretize the partial differential equations. The discretization of each equation yields a set of linear equations, one for each layer, which are solved simultaneously. A modified Gauss elimination procedure is used to solve the tri-diagonal matrix operating only on the three non-zero diagonal terms.

The model is applied in a time step of one hour. The required weather parameters are air temperature, solar radiation (or PAR), wind speed, relative humidity, pond depth and area, fetch, and extinction coefficient. The field data of water temperature profiles with depth are used for calibration of model coefficients.

3. RESULTS AND DISCUSSION

3.1 Calibration

Two ponds were selected for model calibration. Coefficients for wind sheltering (W_{STR}), solar radiation surface absorption fraction (β), and the maximum hypolimnion diffusion coefficient (HK_{max}) were adjusted to achieve an optimal fit between simulations and measurements. The radiation attenuation in the water column (η_o) was also calibrated. Considering the available measured data, the extinction coefficient is varied manually within the range of the measured values. The model calibration was carried out by using a parameter estimation model PEST [Doherty, 1994].

The result of the temperature model calibration run for one pond is given in Figure 1 in comparison with measured iso-thermal lines. The dot signs in the measured contours indicate the locations and timing of the measurements. The regression coefficient (R^2) is 0.95 and 0.91 for the calibration ponds. The maximum over- and under-estimation error is 1.49, -0.52 and 1.82, -0.95 °C for the two ponds. The agreement between measured and simulated temperature is generally good at all depths. There is no indication that a certain depth (surface or bottom) has high or low accuracy in simulations.

The simulation results correctly predicted the time of the actual stratification process events within one hour. The model also accurately predicted the magnitude of the maximum stratification with an error of 0.42 and -0.37 °C for the two calibration ponds, respectively.

3.2 Sensitivity Analysis

The sensitivity analysis for coefficients in the temperature model is made for one of the calibration ponds (reference run). Only one coefficient at a time is changed, and other coefficients are kept at the reference values.

The model is most sensitive to wind speed and solar radiation. High wind speed deepens the mixed layer depth, and results in strong mixing, hence the calculated temperature is much lower at the end of simulation than that of low wind speed. Increasing solar radiation increases the temperature gradient significantly, and decreases the mixed depth. The model is quite sensitive to air temperature and relative humidity. The major effects of these two meteorological parameters are on the surface mixed layer and the mixed depth. There are slight effects on the hypolimnion temperature structure by these two parameters. Varying the epilimnion layer diffusion coefficient (by order of 0.1 and 10) has no effect on

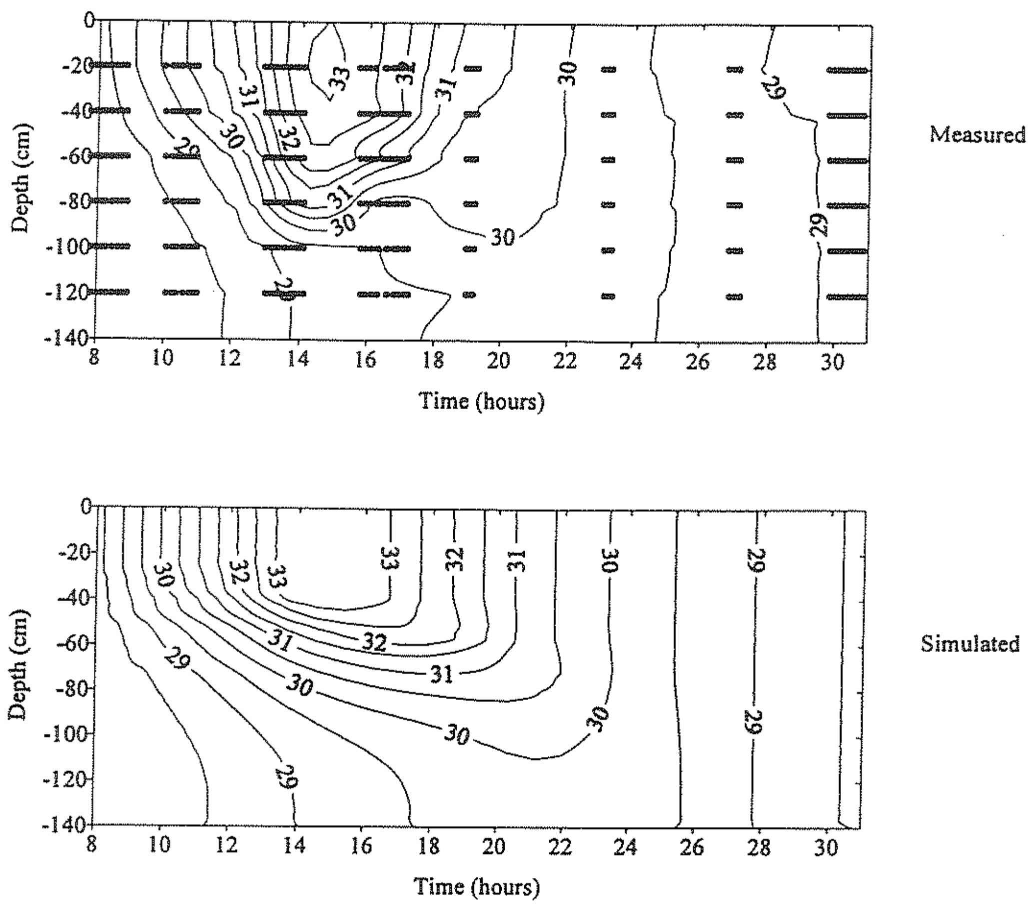

Figure 1 Measured and simulated temperature contours of one of the calibration ponds

the temperature structure at all. This is because the strong wind mixing at the surface mixed layer overrides the diffusion process. Varying the sediment temperature (5 m below water-sediment interface) by 10 °C has no effect on the temperature profiles at all. This is mainly due to the low diffusion coefficient across the interface (in the order of a hundredth of the hypolimnion diffusion coefficient). The model sensitivity to extinction coefficient is in the opposite direction to the maximum hypolimnion diffusion coefficient. Increasing extinction coefficient (similar to reducing the maximum hypolimnion diffusion coefficient) increases the temperature profile gradients. The temperature model is not sensitive to wind sheltering coefficient and solar radiation surface absorption fraction.

It is noted that the timing of the maximum stratification does not vary with the variations of the coefficients in most of the cases.

3.3 Model Validation

After literature review, calibration and sensitivity analysis, the coefficients are fixed in sets depending on the pond environmental conditions. There are no calibration parameters in the final version of the model. The purpose of the model validation is to test the model after generalization.

Five ponds are used for model validation. The regression coefficients (R^2) are larger than 0.76 with an average of 0.84. The maximum under- and over-estimation errors are from -3.14 to 2.73 °C with an average of -1.36 and 1.48 °C, respectively. The model predicts stratification events within 1-3 hours of the actual events. The model also predicts the maximum stratification with maximum errors (under- and over-estimation) from -0.43 to 0.86 °C.

The agreement between the simulated and measured temperature structures is good at all depths for the five validation runs.

4. CONCLUSIONS

Temperature temporal variations and thermal stratification in shallow aquaculture ponds can be accurately simulated using an integral energy model. This approach is an original development for application to aquaculture ponds. The time step of one hour and distance step of 2 cm are sufficient to obtain accurate results. The dynamic, mechanistic model calculates the water temperature of ponds in discrete, homogeneous horizontal layers. Energy exchanges between the pond and atmosphere can be estimated accurately with theoretical and empirical relationships commonly applied to heat balance calculations in other larger and deeper bodies of water. Energy transfer between the layers caused by turbulent mixing is simulated by adopting an integral energy model. Water-sediment energy flux is included in the model. The diffusion coefficient in the epilimnion is computed as a function of wind speed only. The thermocline and hypolimnion diffusion coefficients are related to the Brunt-Vaisala frequency. The model runs for 24 hours.

The temperature simulations match field data well. The temperature model can accurately predict time of stratification onset, maximum stratification and isothermal pond conditions (within 1-3 hours). The model can also accurately predict the maximum stratification magnitude (with errors from -0.43 to 0.86 °C). The model quantifies the pond stratification quite well. The model generally performs well for all depths. The temperature model is most sensitive to solar radiation and wind speed. The model is sensitive to extinction coefficient, the maximum hypolimnion diffusion coefficient, air temperature and relative humidity. The model is insensitive to wind sheltering, radiation surface absorption factor, epilimnion diffusion coefficient and sediment temperature at infinite depth. Water-sediment energy flux can be neglected in modelling practice due to its minor effect on the water column temperature structure. Wind and solar radiation interactions have a significant effect on pond temperature stratification.

ACKNOWLEDGMENTS

The work was funded by CRC for Aquaculture. We would like to thank N. Preston, C. Jackson, M. Burford and M. Austin of CSIRO Marine Laboratories at Cleveland QLD for assisting the data collection program. Special appreciation goes to Noel Herbs and Doug Dark of Gold Coast Marine Prawn Farm for their generous help during the data collection program.

REFERENCES

Bolte, J.P., R.O. Smitherman, and D.T. Hill, 1986, Applications of mathematical modelling to pond aquaculture: a state of the art review, Presented at the 1986 Summer Meeting Am. Soc. Ag. Engineering, San Luis Obsipo, California, June 29-July 2, 1986, ASAE Paper No. 86-5049.

Cathcart, T.P., and F.W. Wheaton, 1987, Modelling temperature distribution in freshwater ponds, *Aquaculture Engineering*, 6: 237-257.

Chang, W.Y.B., and H. Ouyang, 1988, Dynamics of dissolved oxygen and vertical circulation in fish ponds, *Aquaculture*, 74: 263-276.

Cuenco, M.L., 1989, Aquaculture systems modelling: an introduction with emphasis on warmwater aquaculture, International Centre for Living Aquatic Resources Management, Manila.

de Jaeger, J.M., and R.D. Walmsley, 1984, A model to predict water temperature in plastic-covered outdoor mass algal culture systems, *Aquacultural Engineering*, 3: 191-206.

Doherty, J., 1994, PEST: A unique computer program for model-independent parameter optimisation, Proceedings, Water Down Under 94 Conference, Vol. 1, 551-554.

Ford, D.E., and H. Stefan, 1980, Thermal prediction using integral energy model, *J. Hydraul. Div. ASCE*, 106 (HY1): 39-55.

Fritz, J.J., D.D. Meredith, and A.C. Middleton, 1980, Non-steady state bulk temperature determinations for stabilisation ponds, *Water Res.*, 14, 413-420.

Gao, J., and N.P. Merrick, 1996, Simulation of temperature and salinity in a fully mixed pond, *Environmental Software*, vol. 11, Nos. 1-3, pp. 173-178.

Gao, J., 1997, Water quality modelling of aquaculture ponds, PhD Dissertation submitted to the University of Technology, Sydney.

Harleman, D.R.F., 1982, Hydrothermal analysis of lakes and reservoirs, *J. Hydrol. Div. Am. Soc. Civil Eng.*, 108:302-325

Henderson-Sellers, B., 1984, *Engineering Limnology*, Pitman Advanced Publishing Program, Boston, MA, 356 pp.

Hondzo, M., and H.G. Stefan, 1991, Three case studies of lake temperature and stratification response to warmer climate, *Water Resour. Res.*, 27(8): 1837-1846.

Klemetson, S.L., and G.L. Rogers, 1985, Aquaculture pond temperature modelling, *Aquacultural Engineering*, 4: 191-208.

Krant, J.F., F. Motzkin, and H. Gordin, 1982, Modelling temperatures and salinities of mixed seawater fish ponds, *Aquaculture*, 27: 377-388.

Laska, M., 1981, Characteristics and modelling of physical limnology processer, Mitt. Versuchsanst., Wasserbau Hydrol. Glaziol. 54, Zurch, 290 pp.

Losordo, T.M., and R.H. Piedrahita, 1991, Modelling temperature variation and thermal stratification in shallow aquaculture ponds, *Ecological modelling*, 54, 189-226.

Lorsordo, T.M., 1988, Characterization and modelling of thermal and oxygen stratification in aquaculture ponds, Ph.D. Dissertation, University of California, Davis.

Lund, P.D., and R.S. Keinonen, 1984, Radiation transmission measurements for solar ponds, *Solar Energy*, 33:237-240

Marjanovic, N., and G.T. Orlob, 1986, Modelling the hydromechanical and water quality responses of aquaculture ponds, A Literature Review, In Lannan, J.E., Smitherman, R.O. and Tchobanoglous, G. (eds), Principles and Practices of Pond Aquaculture, Oregon State University Press, Corvallis, OR, pp. 207-230.

Octavio, K.A., G.H. Jirka, and D.R.F. Harleman, 1977, Vertical transport mechanisms in lakes and reservoirs, Tech. Rep. 227, Ralph M. Parsons Laboratory, Massachusetts Institute of Technology, Cambridge, MA, 131 pp.

Orlob, G.T., 1981, Models for stratified impoundments, In: A.K. Biswas (Editor), Models for Water Quality Management, McGraw-Hill, New York, pp.273-313.

Orlob, G.T., 1983, One-dimensional models for simulation of water quality in lakes and reservoirs, In: G.T. Orlob (Editor), Mathematical Modeling of Water Quality: Streams, Lakes, and Reservoirs, Wiley, New York, pp.227-273.

Piedrahita, R.H., 1988, Introduction to computer modelling of aquaculture pond ecosystems, *Aquaculture and Fisheries Management*, 19: 1-12.

Piedrahita, R.H., 1991, Modelling water quality in aquaculture ecosystem, In Brune, D.E., J.R. Tomasso (eds), Aquaculture and Water Quality, Advances in World Aquaculture, 3, World Aquaculture Society, Baton Rouge, LA, pp. 322-362.

Rabl, R., and C.E. Nielsen, 1975, Solar ponds for space heating, Solar Energy, 17: 1-12.

Riley, M.J., 1988, User's manual for the dynamic lake quality simulation model MINLAKE. External memorandum No. 213, St. Anthony Falls Hydraulic Laboratory, University of Minnesota, MPLS, MN.

Stefan, H.G., J.J. Cardoni, F.R. Schiebe, and C.M. Cooper, 1983, Model of light penetration in a turbid lake, *Water Resour. Res.*, 19: 109-120.

Wetzel, R.G., 1983, Limnology, Saunders, Philadelphia, PA, 767 pp.

WRE, 1968, Prediction of thermal energy distribution in streams and reservoirs, Report prepared for Department of Fish and Game, State of California, August, Water Resources Engineers, 90pp.